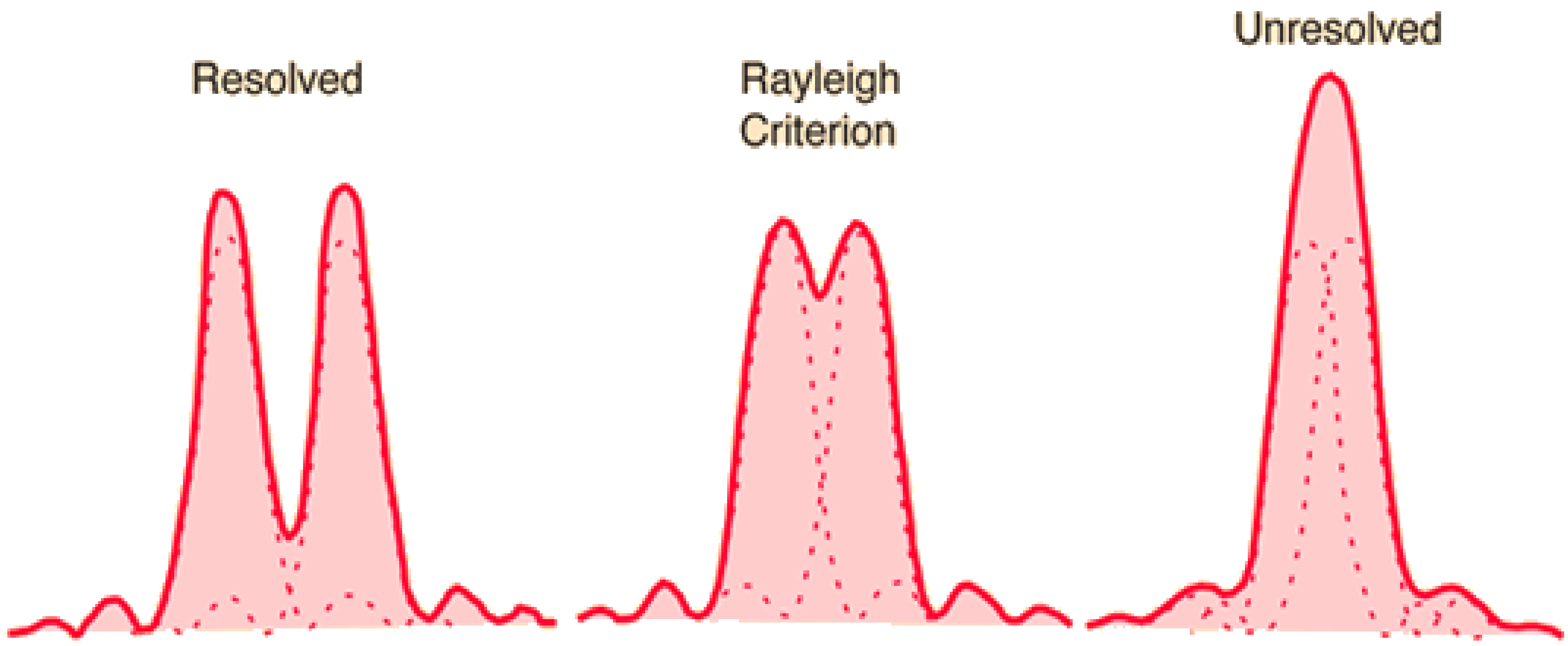
Rayleigh’s Criterion and Resolving Power of Grating

The Rayleigh Criterion

The Rayleigh criterion is the generally accepted criterion for the minimum resolvable detail - the imaging process is said to be diffraction-limited when the first diffraction minimum of the image of one source point coincides with the maximum of another.


Resolved
Rayleigh
Criterion
Unresolved

Single slit

$$\sin\theta_R = \frac{\lambda}{d}$$

Circular aperture

$$\sin\theta_R = 1.22\,\frac{\lambda}{d}$$

If all parts of an imaging system are considered to be perfect, then the resolution of any imaging process will be limited by diffraction.

Considering the single slit expression above, then when the wavelength is equal to the slit width, the angle for the first diffraction minimum is 90°.

This means that the wave is spread all the way to the plane of the slit and will not contain resolvable information about the source of the wave.

This leads to the simplified statement that the limit of resolution of any imaging process is going to be on the order of the wavelength of the wave used to image it.

Dispersion

In order to distinguish different wavelengths that are close to each other a diffraction grating must spread out the lines associated with each wavelength. Dispersion is the term used to quantify this and is defined as

$$D = \frac{\Delta\theta}{\Delta\lambda}$$

Δθ is the angular separation between two lines that differ by Δλ. The larger D the larger the angular separation between lines of different λ.

It can be shown that

$$D = \frac{m}{d\cos\theta}$$

and D gets larger for higher order (m) and smaller grating spacing (d)

Resolving Power

To make lines that whose wavelengths are close together (to resolve them) the line should be as narrow as possible

The resolving power is defined by
$$R = \frac{\lambda_{avg}}{\Delta\lambda}$$

where λ_{avg} is the average of the two wavelengths studied and $\Delta\lambda$ is the difference between them. Large R allows two close emission lines to be resolved

It can be shown that
$$R = Nm$$

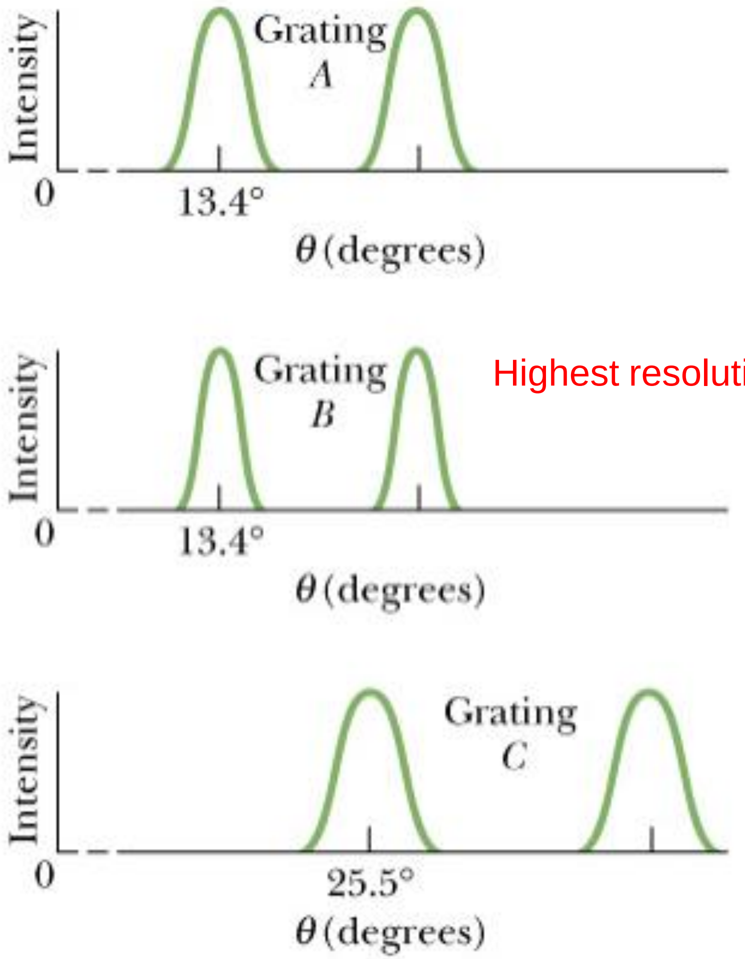
To get a high resolving power we should use as many rulings.

Dispersion and Resolving Power

Three gratings illuminated with light of
λ=589 nm, m = 1

Grating	N	d(nm)	θ	D(°/μm)	R
A	10000	2540	13.4	23.2	10000
B	20000	2540	13.4	23.2	20000
C	10000	1360	25.5	46.3	10000

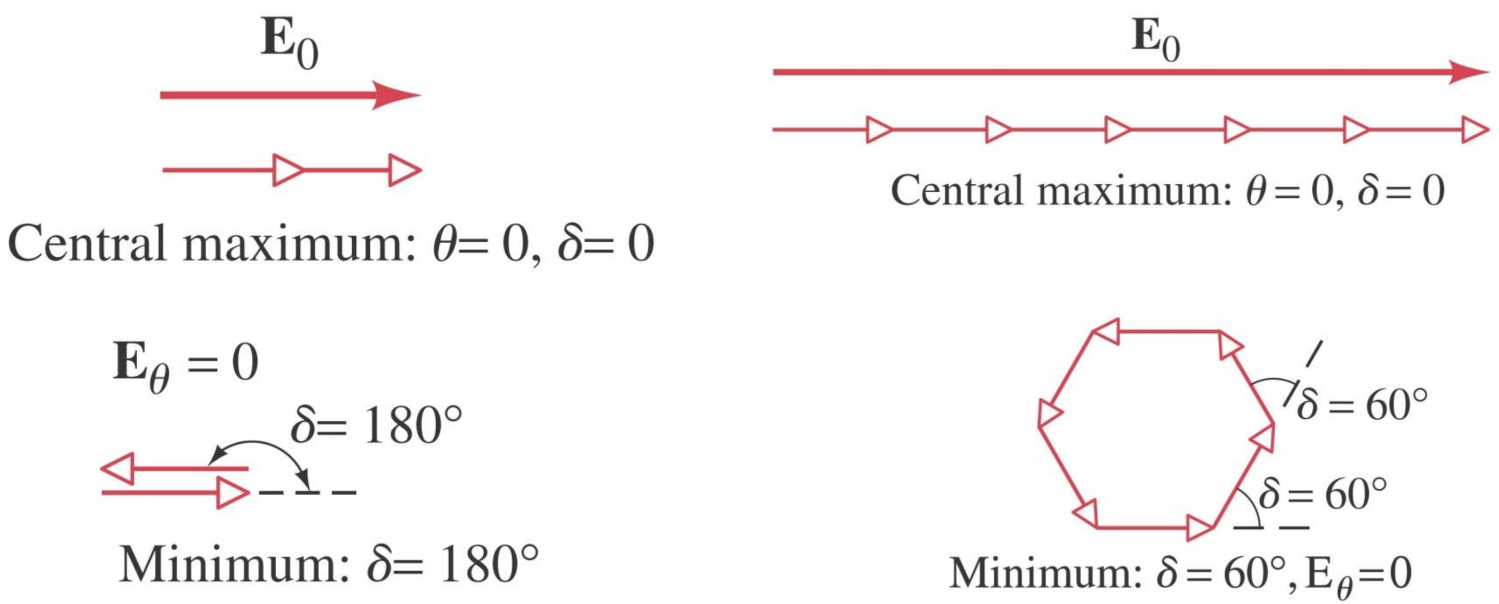
$$R = Nm \qquad D = \frac{m}{d\cos\theta}$$



Highest resolution

Highest dispersion

These two sets of diagrams show the phasor relationships at the central maximum and at the first minimum for gratings of two and six slits.



As the number of slits becomes large, the width of the central maximum becomes very narrow:

$$\Delta\theta_0 = \frac{\lambda}{Nd}.$$

The resolving power of a diffraction grating is the minimum difference between wavelengths that can be distinguished:

$$R = \frac{\lambda}{\Delta\lambda} = Nm.$$